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THE FEASIBILITY OF WALL PARAMETER CORRELATIONS FOR INTERNAL, FIELD-FREE PLASMA FLOWS

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Abstract—A single-fluid theory for the laminar flow of a plasma in the field-free region of a circular tube has been formulated and solved numerically. Through subsequent parametric studies, friction factor and Nusselt number correlations have been obtained in terms of independent parameters evaluated both locally and at the tube entrance. From a consideration of entrance and nonequilibrium effects, as well as from a comparison of the computed wall parameters with available data, it is concluded that general wall parameter correlations, such as apply for low and moderate temperature flows, cannot be obtained for field-free, plasma flows. Whether these correlations are developed from numerical or experimental considerations, they are so closely connected to geometric and entrance effects so as to have little or no general applicability.

NOMENCLATURE

c_p^+, dimensionless specific heat, $c_p/c_{p,r}$;
D, tube diameter;
$D_{amb.}$, ordinary ambipolar diffusion coefficient;
$D^T_{amb.}$, thermal ambipolar diffusion coefficient;
f, friction factor;
h^+, dimensionless enthalpy, $h/c_{p,r}T_r$;
k, Boltzmann constant;
m_A, neutral atom mass;
M_r, Mach number, $u_r/(\gamma_r R T_r)^{\frac{1}{2}}$;
n, particle number density;
$NMAX$, number of radial mesh points used in the numerical solution;
Nu_1, Nusselt number defined in terms of the local temperature potential, $q_w D/k_m(T_w - T_m)$;
Nu_2, Nusselt number defined in terms of the local enthalpy potential, $q_w D c_{p,m}/k_m(h_w - h_m)$;
Nu_3, Nusselt number defined in terms of the temperature potential at the tube entrance, $q_w\, D/k_m(T_w - T_m)_0$;
p, static pressure;
P, pressure defect, $(p_0 - p)/(\rho_r u_r^2)$;
Pr_r, Prandtl number evaluated at reference conditions, $c_{p,r}\mu_r/k_r$;
P_R, plasma radiation;
P_R^+, dimensionless plasma radiation, $2r_0^2 P_R/k_r T_r$;
q_w, wall convective heat flux;
q^+, dimensionless heat flux, $r_0 q_w/k_r T_r$;
r, radial coordinate;
r_0, tube radius;
r^+, dimensionless radius, r/r_0;
R, gas constant;
Re_m, local Reynolds number, $2(\rho u)_m r_0/\mu_m$;
s, a geometrical parameter used in the Skrivan and Von Jaskowsky correlation, equation (21);
T, temperature;
T_{red}, reduced temperature, $(T_w - T)/(T_w - T_m)$;

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u, axial velocity component;
u_m, average axial velocity, $\int_0^r ur^+ \, dr^+$;
u^+, dimensionless axial velocity component, u/u_r;
u_{red}, reduced velocity, u/u_m;
v, radial velocity component;
v^+, dimensionless radial velocity, $(Re_r Pr_r v)/u_r$;
$\bar{V}_j$, species diffusion velocity;
x, axial coordinate;
x^+, dimensionless axial coordinate, $x/(r_0 Re_r Pr_r)$;
$\bar{x}$, dimensionless axial length, $\ln(x/D)$.

Greek symbols
γ, ratio of specific heats;
ρ^+, dimensionless mass density, ρ/ρ_r;
μ^+, dimensionless viscosity, μ/μ_r;
λ^+, dimensionless thermal conductivity, λ/λ_r;
λ_R, the reactive component of the thermal conductivity;
τ_w, wall shear stress;
τ_w^+, dimensionless wall shear stress, $\tau_w r_0/\mu_r u_r$.

Subscripts
A, refers to a neutral atom;
e, refers to an electron;
j, general species designation;
m, properties evaluated at the temperature corresponding to the local mixed mean enthalpy;
o, properties or conditions at the tube entrance;
r, conditions and properties evaluated at the arbitrarily selected reference temperature, $T_r = 14000°K$, and velocity, $u_r = 30000$ cm/s;
w, conditions at the tube wall.

INTRODUCTION

FOR SEVERAL decades, there has been considerable interest in the thermal and hydrodynamic characteristics of internal fluid flows. In particular, knowledge of the heat transfer and shear force at the constraining wall has been important to those concerned with the design of internal flow devices. These quantities are frequently expressed in dimensionless form as the Nusselt number and friction factor, the so-called wall parameters. To facilitate design calculations, much of the effort in this area has been directed towards obtaining general, algebraic correlations for the wall parameters in terms of appropriate independent parameters. Numerous correlations which have been developed for low and moderate temperature flows are discussed by Kays [1].

In recent years, a great deal of interest in the internal flow of high temperature plasmas has evolved from the present or anticipated development of thermonuclear reactors, magneto-gasdynamic energy conversion systems, and supersonic wind tunnels. Plasma internal flows are also used in space vehicle propulsion systems and in several chemical synthesis processes. It is this latter application in particular which has motivated recent studies of field-free plasma flows [2, 3]†. Typically, such a flow condition is produced by passing a gas parallel to a wall-stabilized arc, from whence it emerges as a high temperature plasma (Fig. 1). Flow in the field-free region is subsequently dominated by convective and radiative cooling to the tube wall.

Since an understanding of wall friction and heat transfer effects is desirable for further developments in the area of plasma propulsion and chemical synthesis, several investigators have attempted to extend to the field-free flow of a plasma the wall parameter correlations which have been successfully used in the description of relatively low temperature flows [4–9]. In mounting a calorimeter downstream of a constricted tube arc, Cann [4, 5] was able to correlate the Nusselt number in terms of the Reynolds number for argon, helium, ammonia

† The term "field-free" is used to designate those plasma flows characterized by the absence of external electric or magnetic fields (in contrast to flows which characterize systems such as constricted arcs or $J \times B$ devices).

and hydrogen plasmas. For the most part the heat transfer measurements used to obtain the correlations were large-scale averages taken

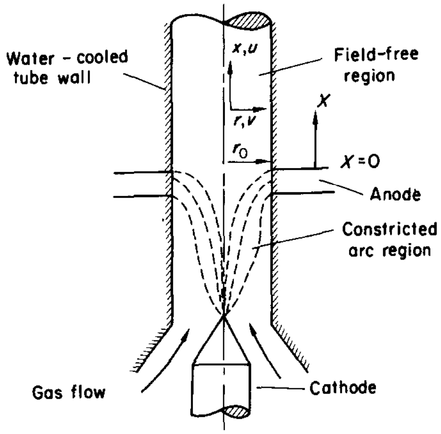

FIG. 1. Schematic of the tube flow physical model.

with a calorimeter length-to-diameter ratio of approximately 8. Similar measurements were made by Wethren and Brodkey [6], who attempted to correlate the Nusselt number in terms of the Graetz number and the friction factor in terms of the Reynolds number for a helium plasma. Skrivan and Von Jaskowsky [7] and Johnson [8] both made heat transfer measurements using a tube comprised of stacked, individually water-cooled segments, each having a length-to-diameter ratio of approximately 1. Hence, they were able to obtain Nusselt number correlations in terms of the axial location for argon, helium, nitrogen and hydrogen. In contrast to the aforementioned experimental investigations, Incropera and Leppert [9] computed the field-free plasma flow characteristics using numerical procedures. In their study, no effort was made to obtain suitable wall parameter correlations or to compare the computed results with experiment.

The objectives of this study were twofold. Using finite-difference methods in conjunction with an equilibrium flow model, parametric studies were performed to obtain suitable correlations for the friction factor and Nusselt number in terms of appropriate independent flow parameters. The availability of such correlations for field-free plasma flows is particularly desirable, since these flows do not readily lend themselves to treatment using "simplified theories", such as those which have been proposed for the asymptotic arc heating region [10]. In the absence of a simplified flow description, one must therefore resort either to a detailed numerical solution or to available correlations. Since the use of detailed numerical procedures to treat each parameter change of interest is undesirable, there is an obvious need for general correlations.

The correlations obtained in this study apply for the undeveloped flow of an argon plasma in a field-free region. Argon was chosen because of the considerable effort which has been expended in recent years to determine its transport properties. Since these properties are now known with considerable accuracy, wall friction and heat transfer may be obtained numerically with some precision.

The second objective is to provide a critical examination of the wall parameter correlations obtained in this and other studies. From the consideration of tube entrance effects and a comparison of the numerical results with available experimental results, several important conclusions are derived concerning the generality and utility of the available correlations.

FLOW MODEL AND SOLUTION

The flow model adopted for this study is similar to that used previously [9], and, when written in terms of dimensionless variables, the governing equations are

$$\frac{\partial}{\partial x^+}(\rho^+ u^+) + \frac{1}{r^+}\frac{\partial}{\partial r^+}(r^+ \rho^+ v^+) = 0 \qquad (1)$$

$$\rho^+\left(u^+\frac{\partial u^+}{\partial x^+} + v^+\frac{\partial u^+}{\partial r^+}\right) = \frac{\mathrm{d}P}{\mathrm{d}x^+} + 2Pr_r\frac{1}{r^+}\frac{\partial}{\partial r^+}\left(r^+\mu^+\frac{\partial u^+}{\partial r^+}\right) \qquad (2)$$

$$\rho^+\left(u^+\frac{\partial h^+}{\partial x^+}+v^+\frac{\partial h^+}{\partial r^+}\right)$$
$$=M_r^2(\gamma_r-1)\left\{2Pr_r\mu^+\left(\frac{\partial u^+}{\partial r^+}\right)^2-u^+\frac{\mathrm{d}P}{\mathrm{d}x^+}\right\}$$
$$+\frac{2}{r^+}\frac{\partial}{\partial r^+}\left(r^+\frac{\lambda^+}{c_p^+}\frac{\partial h^+}{\partial r^+}\right)-P_R^+. \tag{3}$$

In addition the thermal and caloric equations of state are used, and boundary conditions are specified which are consistent with the assumptions of rotational symmetry and zero slip and constant temperature at the wall. The flow is considered to be laminar, and the standard boundary-layer approximations are assumed to apply. Special note should be taken of the fact that this model comprises a single-fluid description of the flow, thereby implying the existence of thermal equilibrium. In light of recent developments [11–13], there is reason to believe that this condition is not satisfied in spatial domains dominated by appreciable plasma cooling effects. The validity of this assumption in the region of the field-free flow directly adjacent to the tube wall is therefore questionable. In this region significant departures from equipartition of translational energy may occur. The influence of this nonequilibrium condition on wall friction and heat transfer will be discussed later in this report.

It should be noted that diffusion and recombination effects are included in the flow model of this study. Due to the appreciable concentration gradients, there will be significant diffusion of ion-electron pairs from the hot plasma core into the cooler regions adjacent to the tube wall. In addition recombination of these pairs into neutral atoms will occur, resulting in the release of the associated ionization energy. Both the diffusion and recombination effects have been included in the model in the manner suggested by Meador and Staton [14]. Briefly, the thermal conductivity for the reacting mixture is written as the sum of a translational component and a reactive component.

$$\lambda=\lambda_t+\lambda_R. \tag{4}$$

The diffusion term, $\sum_j h_j\rho_j\bar{V}_j$, which appears in the expression for the heat flux and includes the recombination energy (h_j includes the chemical enthalpy), is replaced by the term, $-\lambda_R\,\partial T/\partial r$, through appropriate use of the law of mass action and van't Hoff's equation. The resulting expression for the reactive component of the thermal conductivity is

$$\lambda_R=\left\{\frac{nm_A}{2kT^2}\frac{n_en_A}{n_e+n_A}D_{amb.}\,\Delta\tilde{h}+\frac{D_{amb.}^T}{m_AT}\right\}\Delta\tilde{h} \tag{5}$$

where $\Delta\tilde{h}$ is the species enthalpy difference per particle. This expression accounts for the diffusion of electron-ion recombination energy and includes the contributions due to ordinary and thermal ambipolar diffusion. It should also be noted that the inclusion of recombination effects is done subject to the assumption of chemical equilibrium. Therefore, in the presence of non-equilibrium effects, such as are expected near the wall, the above model does not provide an accurate account of recombination effects.

An extensive literature search was conducted to determine the best available transport coefficients for argon. From a comparison of the results of numerous analytical and experimental studies, those by Devoto [15] appeared to be most accurate. In using a Chapman–Enskog solution to the Boltzmann equation, Devoto computed the thermal conductivity and viscosity to the fourth and second approximation, respectively, retained 0(1) terms in the use of charged particle collision cross sections, and in general was meticulous in his selection of available cross section data. Devoto [16] has estimated that his results for viscosity and thermal conductivity are accurate to within 5–15 per cent over the temperature range of interest in this study ($5000 < T < 19000$°K). The thermodynamic properties are those computed by Drellishak, Knopp and Cambel [17], and the dependence of plasma radiation on temperature is obtained from the results due to Evans and Tankin [18], who accounted for both ultra-violet radiation and self-absorption effects.

The flow profiles are obtained from a solution of the governing equations using finite-difference methods discussed previously [9]. The numerical scheme has been made to exhibit the desired stability and convergence characteristics. Typical velocity and temperature profiles are shown in the three-dimensional plots of Figs. 2 and 3. The results are plotted in terms of the reduced velocity and temperature for the extreme conditions of this study, and they illustrate a strong tendency for the profiles to approach a fully developed condition.

Special care was taken in the choice of methods to compute the wall shear stress and heat transfer. Originally, a variety of slope techniques was used to evaluate the gradients in velocity and enthalpy at the tube wall. However, the results obtained from these methods were found to be extremely sensitive to the radial mesh size, despite the fact that the velocity or enthalpy at a particular mesh point differed by less than 2 per cent for a fourfold change in the

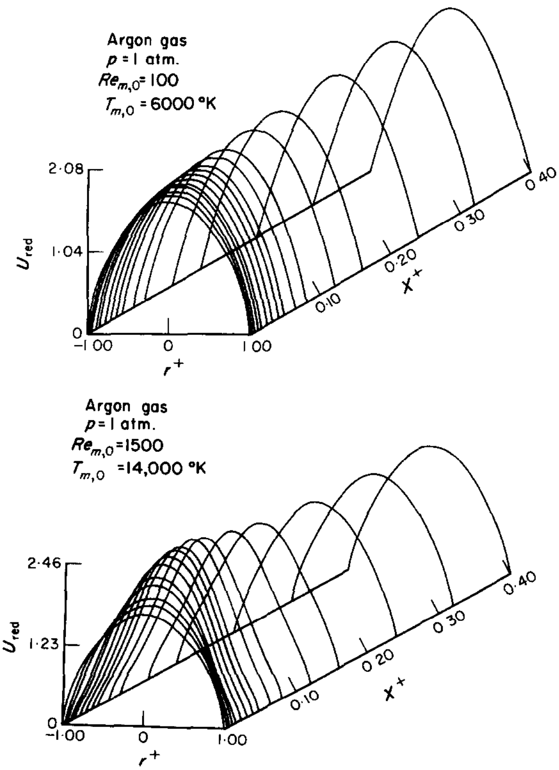

FIG. 2. Three-dimensional plots of the reduced axial velocity for extreme values of the entrance parameters.

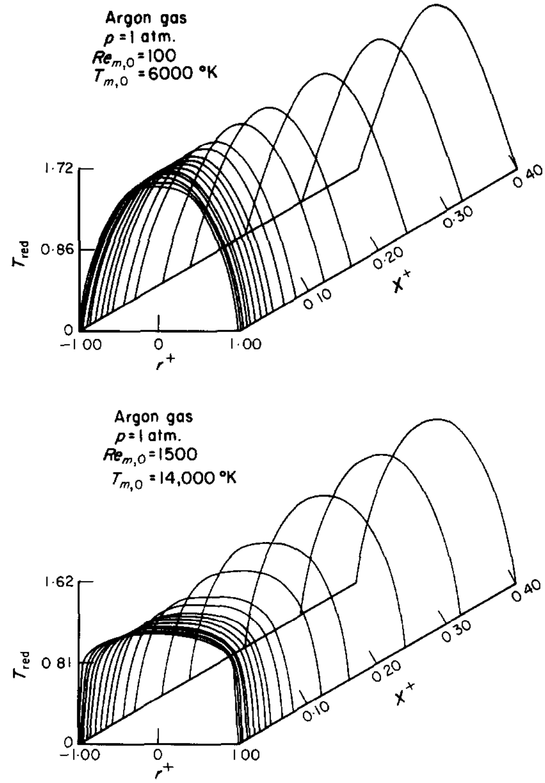

FIG. 3. Three-dimensional plots of the reduced temperature for extreme values of the entrance parameters.

radial mesh size. To circumvent this difficulty, the wall shear stress and heat transfer were computed using momentum and energy conservation equations, respectively. These expressions were obtained by integrating the appropriate partial differential equation from $r^+ = 0$ to $r^+ = 1$ in the transverse direction and from x^+ to $x^+ + \Delta x^+$ in the axial direction. In dimensionless form, the equations are

$$\tau^+_{x^+ + \Delta x^+} = \frac{1}{2Pr_r \Delta x^+} \left\{ \left(\int_0^1 \rho^+ u^{+2} r^+ \, \mathrm{d}r^+ \right)_{x^+ + \Delta x^+} - \left(\int_0^1 \rho^+ u^{+2} r^+ \, \mathrm{d}r^+ \right)_{x^+} \right\} + \frac{1}{4Pr_r \Delta x^+} (P_{x^+} - P_{x^+ + \Delta x^+}) \qquad (6)$$

$$q^+_{x^++\Delta x^+} = \frac{1}{2\Delta x^+}\Bigg\{\left(\int_0^1 \rho^+u^+h^+r^+\,dr^+\right)_{x^++\Delta x^+} - \left(\int_0^1 \rho^+u^+h^+r^+\,dr^+\right)_{x^+} + \tfrac{1}{2}(\gamma_r - 1)M_r^2 \times (P_{x^++\Delta x^+} - P_x)\left[\left(\int_0^1 u^+r^+\,dr^+\right)_{x^++\Delta x^+} - \left(\int_0^1 u^+r^+\,dr^+\right)_{x^+}\right] - (\gamma_r - 1)M_r^2\,Pr_r \times \left[\left(\int_0^1 \mu^+\left(\frac{\partial u^+}{\partial r^+}\right)^2 r^+\,dr^+\right)_{x^++\Delta x^+} + \left(\int_0^1 \mu^+\left(\frac{\partial u^+}{\partial r^+}\right)^2 r^+\,dr^+\right)_{x^+}\right]\Delta x^+ + \left[\left(\int_0^1 P_R^+r^+\,dr^+\right)_{x^++\Delta x^+} + \left(\int_0^1 P_R^+r^+\,dr^+\right)_{x^+}\right]\frac{\Delta x^+}{2}\Bigg\}. \quad (7)$$

The results obtained using a cubic wall slope technique and the above conservation equations are shown in Figs. 4 and 5. Whereas the friction factor and the Nusselt number results obtained from the momentum and energy balance techniques, respectively, are independent of the number of radial mesh points ($NMAX$), the same quantities are extremely sensitive to mesh size when computed using the wall slope method. It is encouraging to find that the results obtained from the slope technique approach those obtained from the balance method in the limit as the number of radial mesh points becomes large. The sharp decreases in the wall parameters which occur at x^+ equal to 0·001 and 0·01 are due to changes in the radial mesh size imposed on the numerical solution at these points. Positive proof of both the suitability of the conservation method and the convergence of the numerical scheme was demonstrated when the results obtained from their application were found to corroborate the friction factor data of Runstadler [19] to within 5 per cent for the asymptotic region of a constricted arc [20].

Before equations (1)–(3) were used to compute the flow field, it was necessary to select appropriate forms of the entrance velocity and enthalpy profiles. Therefore, as a first step in considering the generality of the desired correlations, calculations were performed to determine the effect of a change in the entrance profiles on the wall parameters. Four distinct entrance profile shapes were considered, the linear, parabolic, and cubic profiles, as well as profiles which are representative of conditions in the asymptotic region of a constricted arc. The arc-region profiles were determined by Bower [20] using a numerical scheme similar to that used in this study. It is apparent from the results of this study (Figs. 6 and 7) that the Nusselt number and friction factor are very sensitive to the form of the entrance enthalpy and velocity profiles, respectively, over a distance of up to six tube diameters from the entrance. The obvious conclusion is that any correlation which one might propose, whether it be predicated upon experimental or numerical data, is dependent upon the flow conditions at the exit plane of the device used to produce the field-free flow. This is a result which one must certainly be cognizant of, either in using an existing correlation or in presenting a new correlation.

Fortunately, the field-free conditions of interest in this study can only be produced using a constricted arc facility. In addition, since a frequent design objective is to maximize the mixed mean enthalpy at the entrance to the field-free region, the arc region will most likely be extended to insure the existence of an asymptotic condition. In contrast, there is little physical basis for using the linear or parabolic entrance profiles. On the basis of

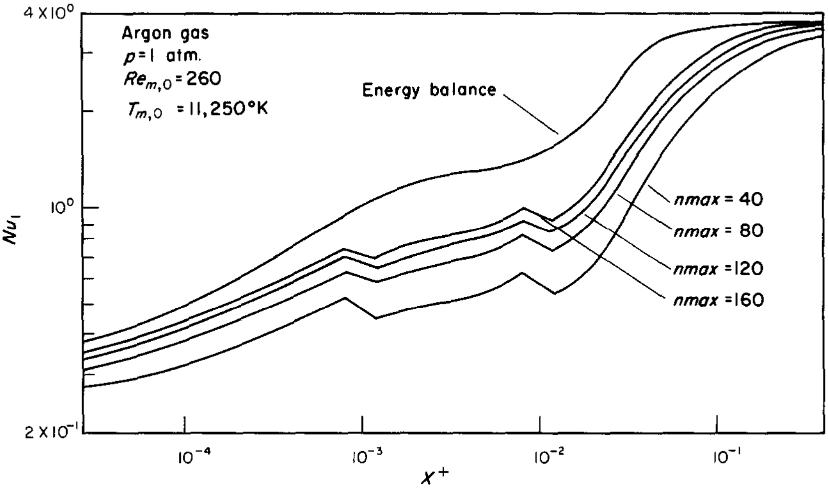

FIG. 4. Comparison of Nu_m vs. x^+ for the wall slope and energy balance methods of computing wall heat transfer.

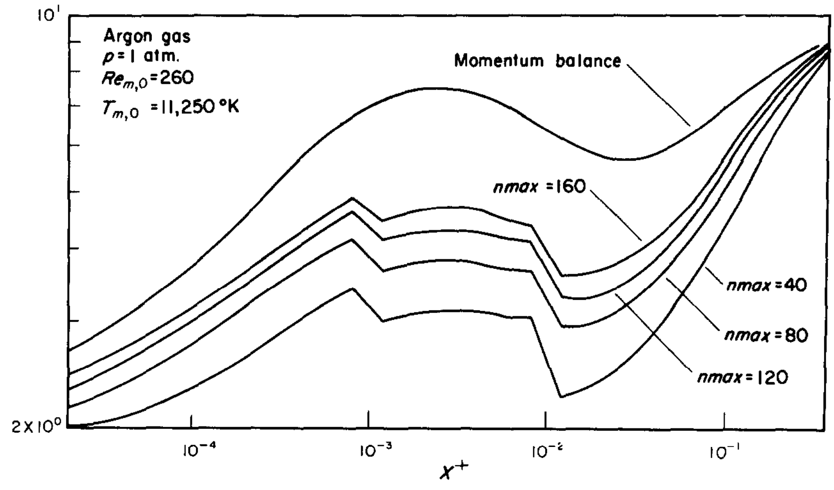

FIG. 5. Comparison of $f.\ Re_m$ vs. x^+ for the wall slope and momentum balance methods of computing wall shear stress.

measurements performed in free plasma jets [21], it appears that the use of cubic profiles would be appropriate if the flow were expanded in passing from the arc to the field-free region. The correlations developed in this study are based upon numerical data obtained using entrance profiles characteristic of conditions in the asymptotic arc region.

WALL PARAMETER CORRELATIONS BASED ON THE NUMERICAL DATA

In order to obtain sufficient wall parameter data for use in determining the correlations, the plasma tube flow calculations were performed for a wide range of entrance conditions. Calculations were performed for entrance Reynolds numbers, $Re_{m,0}$, of 100, 150, 200, 250, 300,

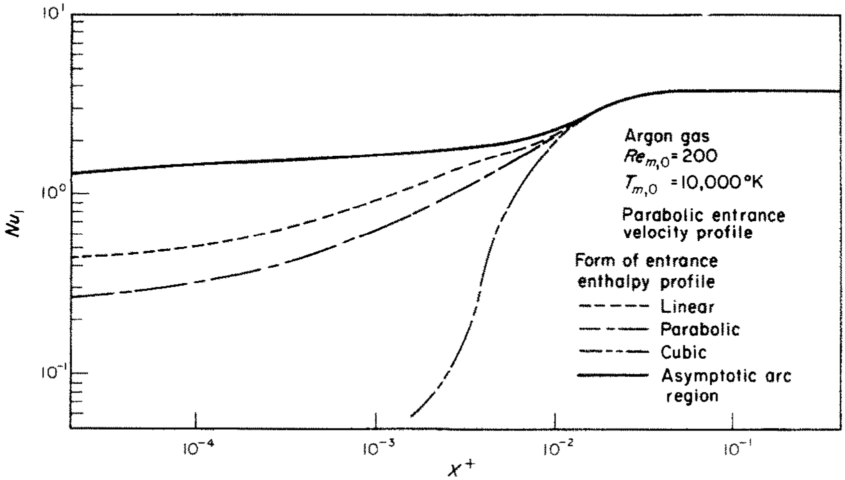

FIG. 6. The effect of entrance enthalpy profile on Nusselt number.

700, 1000 and 1500; for each of these Reynolds numbers, the flow was determined for entrance mixed mean temperatures, $T_{m,0}$, of 6000, 7000, 8000, 10000, 12000 and 14000°K. The wall temperature was maintained at 370°K, and each solution was carried out to an axial location corresponding to $x^{+} = 0{\cdot}3971$ ($x = 58{\cdot}8$ cm). Wall parameters were computed at 22 intervening locations, thereby providing a total of 1046 conditions for use in generating the correlations.

Since the field-free plasma flow is characterized by property values which vary by at least an order of magnitude over the tube cross section, it is perhaps unreasonable to expect wall parameter correlations in terms of the conventional dimensionless parameters. As a

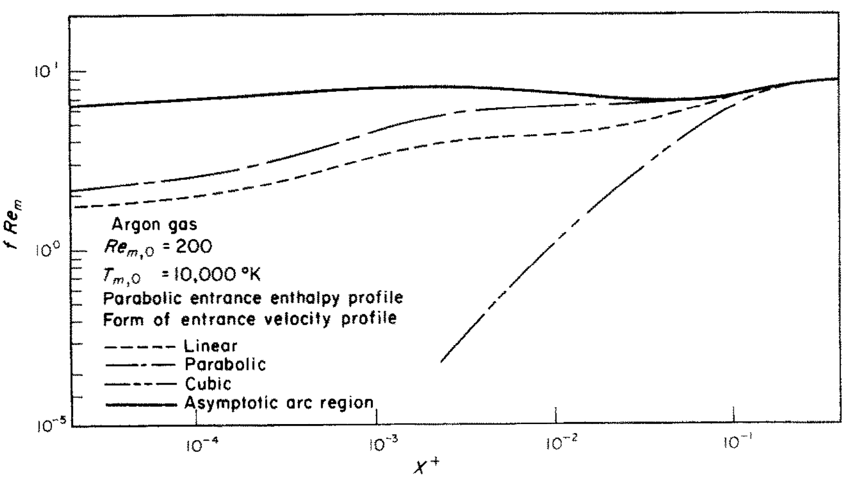

FIG. 7. The effect of entrance velocity profile on friction factor.

first attempt, however, such correlations were sought. In addition to computing f. Re_m, Re_m, Gz_m and T_m/T_w at appropriate axial locations, a total of ten different Nusselt numbers was determined. The definition of the Nusselt numbers varied with the use of an enthalpy potential or a temperature potential, properties evaluated at the wall temperature or the mixed mean temperature, and the use of local or axial average conditions. The Nusselt numbers which were finally selected are those which were most easily correlated. Since the Prandtl number is not a particularly sensitive parameter, it was not used to correlate the numerical data.

Two types of correlations were sought for the friction factor and the various Nusselt numbers. The first type is common to the heat-transfer literature and is simply a correlation of the dependent parameters in terms of the *local* Reynolds number and the mean-to-wall temperature ratio, henceforth to be referred to simply as the temperature ratio. The second type was thought to be of considerable utility to designers working with internal plasma flows and involves a correlation of the wall parameters in terms of the *entrance* Reynolds number and temperature ratio and the axial location. Frequently, these entrance conditions may readily be determined from knowledge of the mass flow rate and the application of a simple energy balance to the arc constrictor.

Upon accumulation of the numerical data, the desired correlations were determined in a systematic fashion. Two different methods were used, one which is termed the matrix method and the other the variational method. Briefly, to illustrate these techniques, consider a correlation of the form

$$Nu = Dx^{+a}Re_{m,0}^{b}(T_m/T_w)_0^c \qquad (8)$$

where a, b, c and D are, as yet, undetermined constants. In taking the logarithm of both sides, there results

$$\ln Nu = \ln D + a \ln x^+ + b \ln Re_{m,0} + c \ln(T_m/T_w)_0. \qquad (9)$$

From the numerical solutions, there are generated a large number, M, of Nusselt numbers and corresponding values of x^+, $Re_{m,0}$, and $(T_m/T_w)_0$. The resulting equations may be written in matrix form as

$$B = AX \qquad (10)$$

where

$$B = \begin{vmatrix} \ln(Nu)_1 \\ \ln(Nu)_2 \\ \cdot \\ \cdot \\ \cdot \\ \ln(Nu)_M \end{vmatrix} \qquad (11)$$

$$A = \begin{vmatrix} 1 & \ln(x^+)_1 & \ln(Re_{m,0})_1 & \ln(T_m/T_w)_{0,1} \\ 1 & \ln(x^+)_2 & \ln(Re_{m,0})_2 & \ln(T_m/T_w)_{0,1} \\ & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ 1 & \ln(x^+)_M & \ln(Re_{m,0})_M & \ln(T_m/T_w)_{0,M} \end{vmatrix} \qquad (12)$$

$$X = \begin{vmatrix} \ln D \\ a \\ b \\ c \end{vmatrix}. \qquad (13)$$

A special subroutine is then used to invert the matrix equation and to solve for X. In effect, the subroutine solves a linear least squares problem in which the Euclidian norm of B–AX is minimized.

The second method requires that two of the independent parameters in equation (8) be held constant while the third is varied over the range of interest. The dependent parameter is then plotted as a function of the independent parameter on a log–log scale, and the exponent of the independent parameter is determined from the slope of the plot. This procedure is used for each of the three independent parameters to obtain values of a, b and c. The coefficient D is then determined by solving (8) for all of the appropriate numerical data and selecting a particular value or function which minimizes the dispersion in the computed results. All of the scanning, calculating and plotting which pertain to the application of the variational method is done on the computer.

Correlations of the Nusselt number in terms of the local Reynolds number and the temperature ratio were attempted for several different Nusselt numbers using the matrix approach. The most successful correlation was obtained for a Nusselt number, Nu_1, defined in terms of the mean thermal conductivity and a temperature potential.

$$Nu_1 = 3{\cdot}80; (T_m/T_w) \leqslant 18$$

$$Nu_1 = 1807\, Re_m^{0{\cdot}11}(T_m/T_w)^{-2{\cdot}33}; \quad 18 < (T_m/T_w) \leqslant 37{\cdot}8. \tag{14}$$

Selected numerical data and the correlation expressed by (14) are plotted in Fig. 8. The important features to note are that the Nusselt number is, to an excellent approximation, independent of Reynolds number and temperature ratio for $(T_m/T_w) < 18$. The significant change in the form of the correlation which occurs at a temperature ratio of 18 is due to the fact that the influence of ionization on the thermodynamic and transport properties becomes appreciable at this point. Note also that the value of the Nusselt number in the low temperature range is very close to the value of 3·66 which pertains to constant property flow characterized by fully-developed velocity and temperature profiles. The expressions given by (14) correlate 80 per cent of the numerical data to within 10 per cent and all of the data to within 30 per cent.

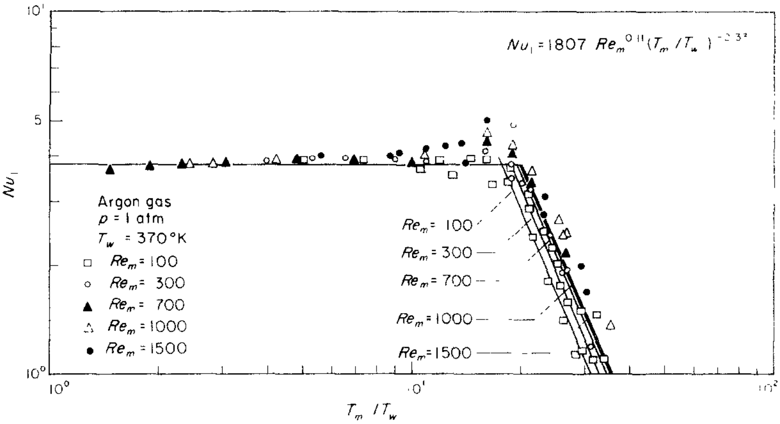

FIG. 8. Correlation for Nusselt number, Nu_1, in terms of local Reynolds number and mean-to-wall temperature ratio.

A similar study resulted in a suitable correlation for the Nusselt number, Nu_2, defined in terms of the mean thermal conductivity and an enthalpy potential.

$$Nu_2 = 3{\cdot}80; (T_m/T_w) \leqslant 18$$

$$Nu_2 = 1{\cdot}65 Re_m^{0{\cdot}115} (T_m/T_w)^{0{\cdot}076}; \quad 18 < (T_m/T_w) \leqslant 37.8. \tag{15}$$

For $(T_m/T_w) < 18$, Nu_2 is also approximately equal to 3·80, however in the high temperature region, this Nusselt number is now only weakly dependent upon the temperature ratio. Again, equation (15) correlates the numerical data to within 30 per cent and reflects a weak dependence on the Reynolds number.

Difficulties were experienced in attempting a similar correlation for the friction factor–Reynolds number product. The following expression was obtained from the matrix method

$$f \cdot Re_m = 11{\cdot}3\,(T_m/T_w)^{-0{\cdot}2} \qquad (16)$$

however, as shown in Fig. 9, the correlation is only adequate for $(T_m/T_w) < 15$. All efforts to explain the erratic, high-temperature behavior in terms of the local Reynolds number and temperature ratio have been unsuccessful.

A consideration of entrance parameter correlations revealed that Nu_2 is the Nusselt number definition which is most amenable to this type of correlation. By application of the variational method to the numerical data, the following expression is obtained

$$Nu_2 = 3{\cdot}5x^{+b}\,Re_{m,0}^{0{\cdot}08}\,(T_m/T_w)_0^{-0{\cdot}10} \qquad (17)$$

where

$$b = 0{\cdot}01\left\{1 - \frac{(T_m/T_w)_0 - 16{\cdot}2}{16{\cdot}2}\right\}. \qquad (18)$$

In addition, by application of the matrix method to the numerical data, the following correlation is obtained

$$Nu_2 = 4{\cdot}59x^{+[0{\cdot}331 - 0{\cdot}028(T_m/T_w)_0 + 0{\cdot}00054(T_m/T_w)_0^2]}\,Re_{m,0}^{0{\cdot}0126}\,(T_m/T_w)_0^{-0{\cdot}0826}. \qquad (19)$$

A comparison of the above correlations with the numerical results is shown in Fig. 10 for three sets of entrance parameters, and in general the results are unsatisfactory. Under conditions of high entrance temperature and Reynolds number, both equations (17) and (19) underpredict the numerical data by as much as 100 per cent near the tube entrance. In addition, both correlations fail to reflect the characteristic Nusselt number development for moderate to high values of $T_{m,0}$ and $Re_{m,0}$ (the sharp decrease in Nu_2 at the entrance followed by a minimum and a gradual rise to some value close to the value for constant-property, fully developed flow). The correlation given by (17) also represents the numerical results poorly at points far downstream from the entrance, where typically the latter is overpredicted by as much as 45 per cent. If an entrance parameter correlation must be used, equation (19) is preferable, however, in general, the correlations given in terms of the local parameters (14) and (15) are felt to be superior.

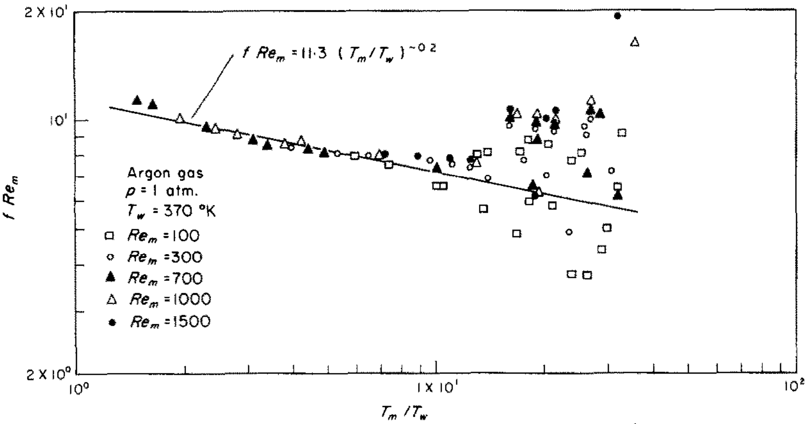

FIG. 9. Correlation for friction factor–Reynolds number product in terms of the local mean-to-wall temperature ratio.

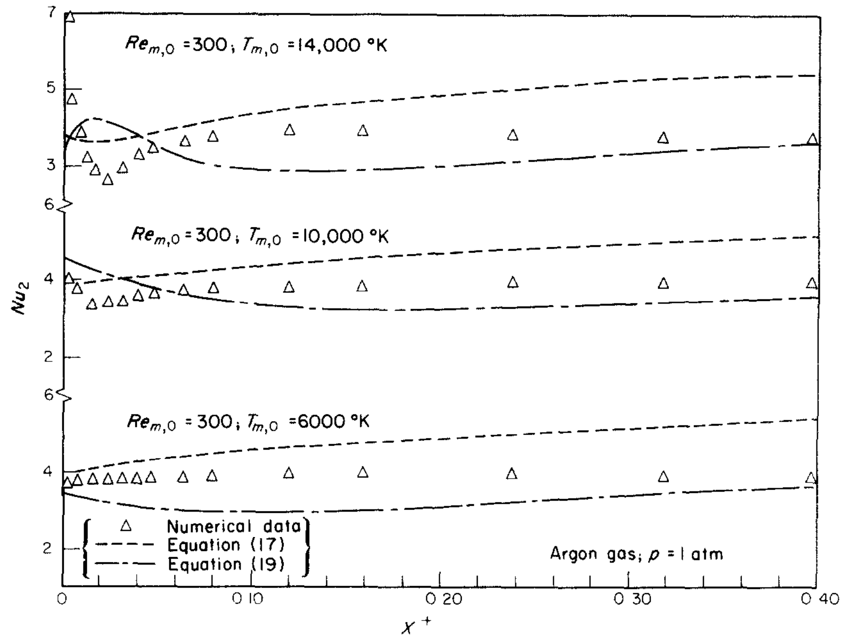

FIG. 10. Comparison of the entrance parameter correlations for Nusselt number, Nu_2, with selected numerical data.

COMPARISON OF THE EQUILIBRIUM FLOW CORRELATIONS WITH EXPERIMENTAL DATA

In this section the correlations derived from the numerical data are compared with the experimental results of other investigators. Although all of the measurements fail to delineate between radiative and convective contributions to the wall heat flux and some are large-scale averages, such comparisons will aid in determining the appropriateness of wall parameter correlations and in suggesting the kind of experimental study which might lead to further clarification.

Cann [4] performed heat transfer measurements using a flow configuration much like the one considered in this study. Successive 2 in. long, water-cooled cylinders were mounted down-stream of the arc and used as calorimeters to measure an axial average wall heat flux. Measurements were made using both argon and helium with mixed mean temperatures as high as 10500°K and an estimated experimental uncertainty of 10 per cent. Unfortunately, Cann reports that an axial magnetic field of approximately 1000 Gauss was necessary to stabilize the arc. Although Cann [4] and Raelson [22] indicate that such a field has a negligible effect on wall heat transfer, recent measurements by Yuen [23] show that the $J \times B$ force resulting from the application of a magnetic field acts as a destabilizing mechanism. Hence, it is possible that the resulting helical instabilities could have an important effect on heat transfer.

Since Cann presented his heat-transfer data in terms of the Nusselt number, Nu_2, a rough comparison with equation (15) is possible (Fig. 11). Despite the fact that there is a sharp contrast between the suggested Reynolds number dependency, the general agreement in magnitudes is good. In addition, the fact that Cann was able to discern no effect of the mean gas temperature on Nu_2 is consistent with the numerical correlation which assigns a small role to this

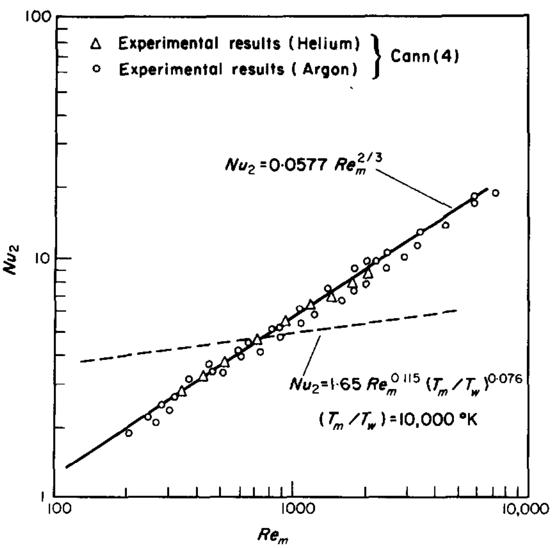

FIG. 11. Comparison of the wall parameter correlation (15) with the experimental results of Cann [4].

quantity only at moderate and high temperatures. Further comparison of these results should be viewed with caution due to the uncertainty concerning the aforementioned instability. However, it should be noted that the experimental results for argon and helium are virtually identical, implying that wall parameter correlations obtained for one monatomic gas are applicable to a different monatomic gas. In considering the wall heat transfer from H_2 and NH_3 plasmas [5], however, it was found that the correlations do not apply to diatomic or polyatomic gases.

Wethren and Brodkey [6] also used an experimental configuration, much like the one considered in this study, to measure the wall shear and heat-transfer characteristics of a helium plasma. Although the heat-transfer measurements are presented in terms of Nu_1, the considerable scatter in their data precludes a direct comparison with equation (14). However, it is of interest to note that, with but two exceptions, the more than 100 data points obtained fall in the Nusselt number range between 0·8 and 8. In fact, the majority of the heat-transfer data is in the Nusselt number range between 2 and 4. The close proximity of these results to the constant property value for fully developed flows, $Nu_1 = 3{\cdot}66$, is consistent with the numerical results of this study and the conclusions of Eckert [24]. In addition, the friction factor data, although also subject to considerable scatter, are clustered about the isothermal condition, $f \cdot Re_m = 16$, a result which is also consistent with the numerical data of this study. Both the numerical and experimental wall parameter results therefore sustain the conclusion derived from Figs. 2 and 3, that is, the field-free plasma flow readily attains a fully developed condition.

Because of their use of large-scale calorimeters, Cann [4, 5] and Wethren and Brodkey [6] were unable to obtain experimental correlations for the wall parameters in terms of the axial coordinate. However, more localized measurements were made by Skrivan and Von Jaskowsky [7] and Johnson, Choksi and Eubank [8], and in both cases correlations were attempted in terms of the axial coordinate. Each investigation used a tube comprised of stacked, water-cooled segments. The Skrivan apparatus was used to provide a tube total length-to-diameter ratio as high as 38, whereas the Johnson arrangement provided a maximum L/D of approximately 6. A unique feature of this experimental work is that, in both cases, the constant diameter field-free region is immediately preceded by a transition section in which the plasma flow is expanded. In one study [7], the plasma from the arc passes through a converging–diverging nozzle, and in the other investigation [8], the gas undergoes an abrupt step expansion upon entering the field-free region. This feature is particularly important since the expansion of a plasma flow is known to provide an inflection point in the profiles [21]. The experimental devices used in the above studies are therefore most likely characterized by cubic velocity and enthalpy profiles at the tube entrance, profiles which are therefore not compatible with those used in the numerical solution. This consideration, plus the likelihood of flow recirculation effects at the tube entrance of the stepped device, is highly relevant to any comparisons which might be made.

Skrivan and Von Jaskowsky [7] define a Nusselt number, Nu_3, in terms of a local heat flux and mean thermal conductivity and a fixed temperature potential which is evaluated at the tube entrance. From consideration of A, N_2 and H_2 plasmas, they arrive at the following correlation for Nu_3 in terms of the axial position and the entrance Reynolds number and mean-to-wall thermal conductivity ratio

$$Nu_3 = 0{\cdot}0095\, Re_{m,0}^{1\cdot 25} \left(\frac{x + s}{D}\right)^{-1\cdot 67} \left(\frac{k_m}{k_w}\right)_0^{0\cdot 4}. \quad (20)$$

The quantity s is a geometrical parameter needed to correlate the data; a value of 0·121 ft was used for calculations performed in this study. Similarly, Johnson [8] obtained a correlation for Nu_1 from measurements performed on He, A and N_2 plasmas. Unlike the Skrivan results, the constants appearing in the Johnson correlation depend upon the type of gas. For argon, the following expression was found to be appropriate

$$Nu_1 = 7{\cdot}7\,\{\exp(-0{\cdot}818\,\bar{x}^2 + 0{\cdot}764\,\bar{x})\} \times Re_m^{0\cdot 517}\left(\frac{\mu_m}{\mu_w}\right)^{-1\cdot 50}. \quad (21)$$

Note that, in both experimental studies, the mixed mean temperature at the tube entrance was never in excess of 7000°K, in which case the radiative contribution to the wall heat flux is negligible.

Due to the extreme differences between the flow configuration considered in this study and the experimental arrangements used by Skrivan and Johnson, a systematic comparison of the corresponding results is most difficult. However, if for no other reason that to provide further substance to the premise than general wall-parameter correlations are nonexistent, such a comparison was made for extreme values of the entrance parameters.

In Fig. 12 the comparison is made for values of the entrance Reynolds number and mean temperature characteristic of the experimental studies. The large disparity in the three sets of results is readily apparent. The difference between the Skrivan correlation and the results of the equilibrium theory are most likely attributed to the existence of different entrance temperature profiles and/or the existence of non-equilibrium effects. The fact that the Skrivan correlation underpredicts the numerical data is consistent with the results of Fig. 6, which reflect a reduction in Nusselt number in passing from the asymptotic arc region profile used in the numerical study to the cubic profile which characterizes the experimental system. However, the fact that the difference between the Skrivan correlation and the numerical results increases with axial position contradicts the results of Fig. 6. A possible explanation may be obtained from consideration of non-equilibrium effects. Both Bahadori and Soo [11] and Jacobs and Grey [12] show that departures from equipartition of translational energy occur in regions of intense plasma cooling. In particular the amount by which the electron temperature exceeds the heavy-particle temperature is shown to increase with both increasing radial and axial position, such behavior being attributed to the "freezing" of the electron temperature at approximately 5000°K while the heavy particle temperature continues to decay. Bahadori and Soo also use a semi-empirical method to show that the non-equilibrium thermal conductivity exceeds the equilibrium value by an appreciable amount with no apparent effect on the Nusselt number. In addition, Powers [25] applied both multi- and single-fluid theories to the prediction of wall heat transfer for a Couette flow and found that the aforementioned non-equilibrium effects act to reduce the wall heat transfer. Hence, it would appear that a Nusselt number based upon an experimentally determined wall heat transfer and an equilibrium thermal conductivity must be less than that computed exclusively from an equilibrium theory. This conclusion is verified from a comparison of the numerical data with the

Skrivan correlation in Fig. 12; that is, if the departure from equilibrium becomes more pronounced with increasing axial location and if this departure reduces the total wall heat transfer, then the difference between the results computed from an equilibrium theory and obtained experimentally should increase in the axial direction. To clarify this matter, a comparison should be made between numerical and experimental results characterized by the same entrance profiles.

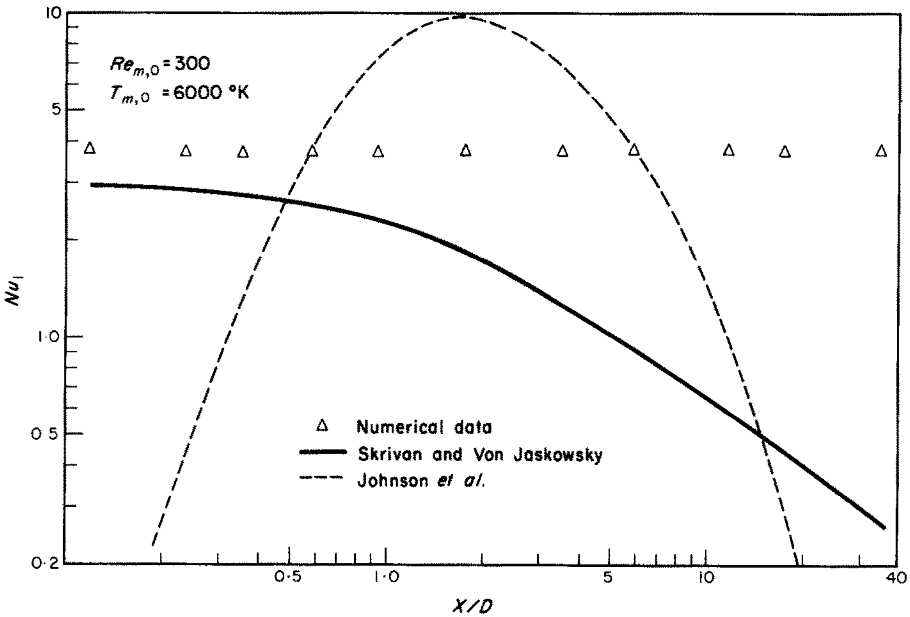

FIG. 12. Comparison of the Skrivan and Von Jaskowsky [7] and Johnson *et al.* [8] experimental correlations with the numerical data for $Re_{m,0} = 300$ and $T_{m,0} = 6000$°K.

The Johnson correlation shown in Fig. 12 is only representative of the experimental data for $2 < x/D < 6$. The fact that it predicts extremely small Nusselt numbers both at the entrance and in the limit of large x further reflects the absence of generality; that is, there is no basis for assuming the correlation to be applicable outside the limited spatial region for which it was obtained. In addition, the fact that the correlation provides results which differ considerably from the numerical data and the Skrivan correlation in the region $2 < x/D < 6$ shows again the importance of entrance effects.

The aforementioned correlations and the numerical data are shown in Fig. 13 for a field-free flow characterized by an entrance mean temperature of 14000°K. Since this exceeds by a factor of two the experimental conditions for which the Skrivan and Johnson correlations were obtained, there is no reason to expect these correlations to provide a valid representation of the Nusselt number at this temperature. The appreciable difference in results serves to further illustrate the difficulty associated with obtaining general heat-transfer correlations.

CONCLUDING REMARKS

An equilibrium model has been formulated for the laminar, field-free flow of a plasma in a circular tube and solved numerically. Parametric studies have been performed to determine general wall parameter correlations for an argon gas. Further consideration of entrance effects and a comparison of the computed wall parameters with available data has led to several conclusions concerning the extent to which such correlations may be used to describe field-free plasma flows.

The most important consequence of this study is that, since a variety of factors influence the wall shear and heat-transfer characteristics, general, wall-parameter correlations, such as apply for low and moderate temperature flows, are nonexistent. Those factors which play an important role in determining thermal and hydrodynamic wall effects may be delineated as follows:

1. The geometrical configuration immediately upstream of the field-free region and therefore the form of the velocity and temperature profiles at the tube entrance strongly influence the downstream wall characteristics (Figs. 6, 7, 12, 13). The person wishing to use an existing correlation must therefore be careful to insure that the entrance conditions associated with the correlation are like the entrance conditions which characterize his intended application. In effect, this sensitivity to the form of the entrance profiles restricts the generality of any wall parameter correlations, obtained numerically, experimentally, or otherwise.

2. Assuming the form of the entrance profiles to be fixed, it is appropriate to question whether a single heat transfer correlation may be used

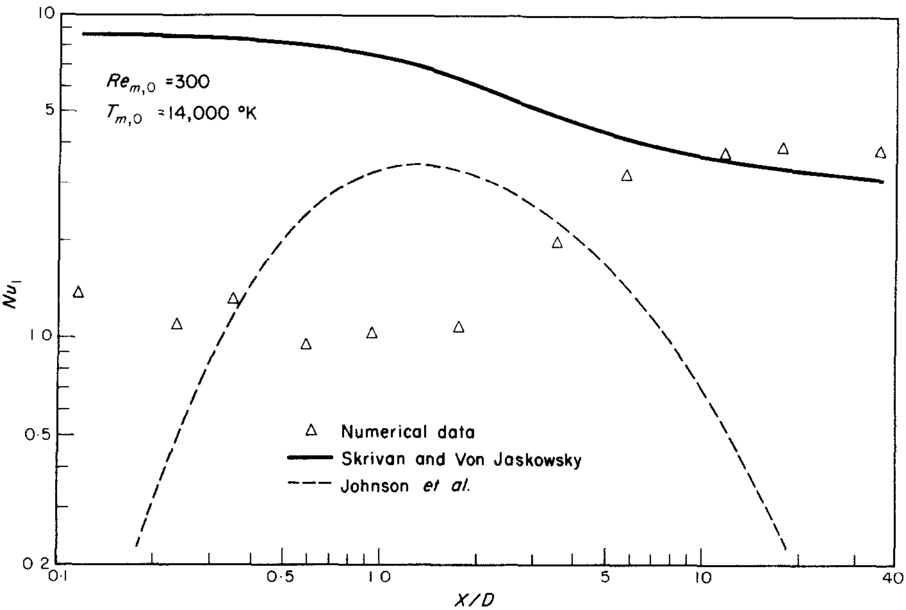

FIG. 13. Comparison of the Skrivan and Von Jaskowsky [7] and Johnson *et al.* [8] experimental correlations with the numerical data for $Re_{m,0} = 300$ and $T_{m,0} = 14000°K$.

for more than one gas. Although the numerical work of this study was restricted to argon, several gases were considered independently in a number of the experimental studies [4, 5, 7, 8]. Although Johnson [8] could not obtain a single correlation which applied for A, He and N_2, Cann [4] obtained one correlation which adequately described heat transfer effects in both A and He plasmas. This correlation failed, however, to describe conditions in NH_3 and H_2 plasmas [5]. Through the use of an appropriate thermal conductivity ratio, however, Skrivan and Von Jaskowsky [7] obtained a single heat transfer correlation for A, N_2 and H_2 plasmas. General correlations therefore appear possible to the extent that, for entrance profiles of a prescribed form, a single correlation may be derived which applies to more than one gas. This would certainly be true for the monatomic gases, all of which are characterized by a similar temperature dependence for the thermodynamic and transport properties.

3. On the basis of the results of Bahadori and Soo [11], thermal nonequilibrium effects are not expected to have any influence on efforts to develop general friction factor correlations. The insensitivity of hydrodynamic effects to nonequilibrium conditions is further substantiated by the excellent agreement between the numerical results obtained by Bower [20], using a single-fluid theory, and the friction factor data acquired by Runstadler [19] for the asymptotic region of a laminar constricted arc. However, the considerable scatter in the numerical data of Fig. 9 about the correlation of equation (16) is perhaps indicative of the fact that, for entrance profiles of the prescribed form, an additional independent parameter is needed to obtain a satisfactory correlation for the friction factor. It appears that the local Reynolds number and wall temperature ratio are insufficient.

4. Unlike the wall shear force, the heat transfer is strongly influenced by the existence of a wall-induced thermal non-equilibrium condition. This fact has been brought out by the comparisons of this study as well as by the work of Bahadori and Soo [11] and Powers [24]. In fact, the existence of a nonequilibrium condition restricts the development of general heat transfer correlations to much the same, if not a greater, extent than the aforementioned geometry and entrance effects. Presently, the manner in which nonequilibrium phenomena affect wall heat transfer is not clearly understood. The results of this study suggest that the matter could be clarified by obtaining heat-transfer data from an experimental facility which provided entrance conditions identical to those used in the present numerical solution. Such an experimental program has been initiated in this laboratory.

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POSSIBILITÉ DE CORRÉLATIONS DU PARAMÈTRE DE PAROI POUR DES ÉCOULEMENTS INTERNES DE PLASMA SANS CHAMP

Résumé—Une théorie à fluide unique pour l'écoulement laminaire d'une plasma dans la région sans champ d'un tube circulaire a été formulée et résolue numériquement. A l'aide d'études paramétriques postérieures, des corrélations pour le coefficient de frottement et le nombre de Nusselt ont été obtenues en fonction de paramètres indépendants évalués à la fois localement et à l'entrée du tube. On conclut, en considérant les paramètres de paroi calculés avec les résultats disponibles, que des corrélations générales de paramètres, telles que celles qui s'appliquent aux écoulements à températures basses et modérées, ne peuvent pas être obtenues pour des écoulements de plasma sans champ. Que ces corrélations soient obtenues à partir de considérations numériques et expérimentales, elles sont reliées si étroitement aux effets géométriques et d'entrée qu'elles ont peu ou aucune applicabilité générale.

Zusammenfassung—Für die Laminarströmung eines Plasmas im feldfreien Bereich eines Kreisrohres wurde eine Einkompenetentheorie formuliert und numerisch gelöst. Durch daran anschliessende Parameterstudien konnten Beziehungen für den Reibungsbeiwert und die Nusseltzahl in Abhängigkeit von unabhängigen Parametern gefunden werden, die sowohl lokal als auch am Rohreintritt ermittelt wurden. Aus einer Betrachtung von Eintritts- und Nichtgleichgewichtseinflüssen, ebenso wie aus einem Vergleich der berechneten Wandparameter mit verfügbaren Messungen, wird geschlossen, dass allgemeine Wandparameterbeziehungen für Strömungen mit niedrigen und mittleren Temperaturen für feldfreie Plasmaströmungen nicht erhalten werden können. Gleichgültig, ob diese Beziehungen aus numerischen oder experimentellen Betrachtungen entwickelt werden, sind sie so eng mit Geometrie und Eintrittseinflüssen verbunden, dass sie wenig oder keine allgemeine Anwendbarkeit besitzen.

Аннотация—В приближении одножидкостной модели сформулирована и численно решена задача ламинарного течения плазмы в свободной от поля области кольцевой трубы. С помощью последующих параметрических исследований получены корреляции коэффициента трения и критерия Нуссельта с использованием независимых чисел, отнесенных как к локальным параметрам, так и к параметрам на входе в трубу. С учетом эффектов на входе и неравновесных эффектов, а также из сравнения рассчитанных параметров у стенки имеющимися данными сделан вывод, что обобщения, отнесенные

к параметрам у стенки, применимые для течений при низких и средних температурах, нельзя получить для свободных от поля течений плазмы. Если такие корреляции получены путем численных решений или обобщений, экспериментальные данные тесно связаны с геометрическими эффектами и эффектами на входе, что их нельзя распространить на более общий случай или же можно это сделать в весьма ограниченной степени.